Use an appropriate sequence or series formula to write the repeating decimal $0.6\overline{54}$ as a simplified fraction. **SCORE: _____ / 15 PTS**

NOTE: Only the 54 is repeated.

$$\frac{6}{10} + \frac{54}{1000} + \frac{54}{100000} + \frac{54}{10000000} + \cdots$$

$$* \frac{1}{100} \qquad * \frac{1}{100}$$

$$= \boxed{\frac{3}{5}} + \boxed{\frac{\frac{54}{1000}}{1-\frac{1}{100}}} = \frac{3}{5} + \frac{\frac{54}{1000}}{\frac{99}{100}} \cdot \frac{1000}{1000} = \frac{3}{5} + \boxed{\frac{54}{990}} = \frac{3}{5} + \frac{3}{55} = \boxed{\frac{36}{55}}$$

2 3 4 2 4

Consider the sequence defined recursively by $a_{n+2} = (n+2)a_n - na_{n+1} - 4$, $a(1) = -17$, $a(2) = -24$. **SCORE: _____ / 20 PTS**

[a] Find the first 6 terms of the sequence. **Your answers must be integers or fractions, NOT decimal approximations.**

$n=1: a_3 = 3a_1 - 1a_2 - 4 = 3(-17) - 1(-24) - 4 = -51 + 24 - 4 = -31$

$n=2: a_4 = 4a_2 - 2a_3 - 4 = 4(-24) - 2(-31) - 4 = -96 + 62 - 4 = -38$

$n=3: a_5 = 5a_3 - 3a_4 - 4 = 5(-31) - 3(-38) - 4 = -155 + 114 - 4 = -45$

$n=4: a_6 = 6a_4 - 4a_5 - 4 = 6(-38) - 4(-45) - 4 = -228 + 180 - 4 = -52$

8

1 $-17, -24, -31, -38, -45, -52$ 6

[b] Based on the first 6 terms, does the sequence appear to be arithmetic, geometric or neither ? Show how you reached your conclusion.

2 ARITHMETIC, $d = -7$ 2

$-17 + (-7) = -24$ $-38 + (-7) = -45$

$-24 + (-7) = -31$ $-45 + (-7) = -52$ 1

$-31 + (-7) = -38$

HJ moved into a rented house, and his contract required him to pay for the water he used. On March 1, 2016, he was charged $39 for water usage. On the 1st day of every month after that, HJ was charged 1.5% more than he was charged on the 1st day of the previous month. By March 2, 2018, how much had HJ been charged for water usage altogether ?

SCORE: _____ / 15 PTS

$$39 + 39(1.015) + 39(1.015)(1.015) + 39(1.015)(1.015)(1.015) + \cdots$$

(25 TERMS)

5

$$= \boxed{\frac{39(1.015^{25} - 1)}{1.015 - 1}}$$ 8

$$= \underline{\$1172.46}$$ 2

Simplify $\binom{6n-3}{6n-7}$. **SCORE: _____ / 15 PTS**

$$\frac{(6n-3)!}{(6n-7)!\,(6n-3-(6n-7))!} = \boxed{\frac{(6n-3)!}{(6n-7)!\,4!}} = \frac{(6n-3)\overset{3n-2}{\cancel{(6n-4)}}(6n-5)\overset{n-1}{\cancel{(6n-6)}}\cancel{(6n-7)!}}{\cancel{(6n-7)!}\,\underset{2}{\cancel{4}}\cdot\cancel{3}\cdot\cancel{2}\cdot 1}$$

4 4 3

$$= \frac{1}{2}(6n-3)(3n-2)(6n-5)(n-1)$$

4

Find a_n for the arithmetic sequence with $a_3 = 2x^2 - 4x$ and $a_8 = 1 - x^2$. **SCORE: _____ / 20 PTS**

$$a_8 = a_3 + 5d$$

$$1 - x^2 = 2x^2 - 4x + 5d \quad 4$$

$$1 + 4x - 3x^2 = 5d$$

$$d = \frac{1}{5}(1 + 4x - 3x^2) \quad 4$$

$$a_3 = a_1 + 2d$$

$$2x^2 - 4x = a_1 + 2 \cdot \frac{1}{5}(1 + 4x - 3x^2) \quad 4$$

$$a_1 = 2x^2 - 4x - \frac{2}{5} - \frac{8}{5}x + \frac{6}{5}x^2$$

$$= \frac{16}{5}x^2 - \frac{28}{5}x - \frac{2}{5}$$

$$= \frac{2}{5}(8x^2 - 14x - 1) \quad 4$$

$$a_n = \frac{2}{5}(8x^2 - 14x - 1) + \frac{1}{5}(1 + 4x - 3x^2)(n-1) \quad 4$$

Use sigma notation to write the series $\frac{2}{19} + \frac{6}{12} + \frac{18}{5} - \frac{54}{2} - \cdots - \frac{13122}{37}$.

← GEOMETRIC $r = 3$

← ARITHMETIC $d = -7$

SCORE: _____ / 15 PTS

1

$$\sum_{n=1}^{9} \frac{2 \cdot 3^{n-1}}{19-7(n-1)}$$

5

5

1

2

$19 - 7(n-1) = -37$

$-7(n-1) = -56$

$n - 1 = 8$

$n = 9$

1 INDEX MATCHES UNDER + INSIDE Σ

Consider the expression $(3x^2 - \frac{2}{x})^{17}$. ② EACH EXCEPT AS NOTED SCORE: _____ / 25 PTS

[a] Write the first 3 terms of the expansion of the expression. Simplify all exponents.

Your answer may use multiplication and exponents, but NOT division, ! nor $_nC_r$ (or equivalent) notation.

$$(3x^2)^{17} + \binom{17}{1}(3x^2)^{16}\left(-\frac{2}{x}\right) + \binom{17}{2}(3x^2)^{15}\left(-\frac{2}{x}\right)^2$$

$$= 3^{17}x^{34} - 17 \cdot 3^{16} \cdot 2x^{31} + 17 \cdot 8 \cdot 3^{15} \cdot 4x^{28}$$

$$\binom{17}{1} = \frac{17!}{1!16!} = \frac{17 \cdot \cancel{16!}}{\cancel{16!}} = 17$$

$$\binom{17}{2} = \frac{17!}{2!15!} = \frac{17 \cdot \overset{8}{\cancel{16}} \cdot \cancel{15!}}{\cancel{2} \cdot 1 \cdot \cancel{15!}}$$

[b] Find the coefficient of x^{-5} in the expansion.

Your answer may use multiplication, division, exponents and !, but NOT $_nC_r$ (or equivalent) notation.

$$\sum_{i=0}^{17}\binom{17}{i}(3x^2)^{17-i}\left(-\frac{2}{x}\right)^i = \sum_{i=0}^{17}\binom{17}{i}3^{17-i}(-2)^i x^{34-2i-i} = \sum_{i=0}^{17}\binom{17}{i}3^{17-i}(-2)^i x^{34-3i}$$

$34 - 3i = -5$

$-3i = -39$

① $i = 13$

$$\binom{17}{13}3^4(-2)^{13} = \boxed{\frac{17!}{13!4!}} ① \, 3^4(-2)^{13} = \frac{17 \cdot \overset{2}{\cancel{16}} \cdot \overset{5}{\cancel{15}} \cdot 14 \cdot \cancel{13!}}{\cancel{13!}\,\cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot 1} ① \, 3^4(-2)^{13}$$

$$= -17 \cdot 5 \cdot 14 \cdot 3^4 \cdot 2^{14}$$

Prove that $a + ar + ar^2 + \cdots + ar^{n-1} = \dfrac{a(r^n - 1)}{r-1}$ for all positive integers n **using mathematical induction**. **SCORE: _____ / 25 PTS**

$\uparrow n \geq 1$

NOTE: Do NOT use a series formula.

BASIS STEP: PROVE $a = \dfrac{a(r^1 - 1)}{r-1}$ $1\frac{1}{2}$

$1\frac{1}{2}$ $\dfrac{a(r^1-1)}{r-1} = \dfrac{a\cancel{(r-1)}}{\cancel{r-1}} = a$

INDUCTIVE STEP: ASSUME $a + ar + ar^2 + \cdots + ar^{k-1} = \dfrac{a(r^k - 1)}{r-1}$ FOR SOME ARBITRARY INTEGER $k \geq 1$ $2\frac{1}{2}$ $2\frac{1}{2}$

$3\frac{1}{2}$ $a + ar + ar^2 + \cdots + ar^{k-1} + ar^k$

$2\frac{1}{2}$ $= \dfrac{a(r^k - 1)}{r-1} + ar^k$

3 $= \dfrac{a}{r-1}\left(r^k - 1 + r^k(r-1)\right)$

3 $= \dfrac{a}{r-1}\left(\cancel{r^k} - 1 + r^{k+1} - \cancel{r^k}\right)$

$2\frac{1}{2}$ $= \dfrac{a}{r-1}\left(r^{k+1} - 1\right) = \dfrac{a(r^{k+1} - 1)}{r-1}$

BY MI, $2\frac{1}{2}$
$a + ar + ar^2 + \cdots + ar^{n-1} = \dfrac{a(r^n - 1)}{r-1}$ FOR ALL INTEGERS $n \geq 1$